SE 2AA4, CS 2ME3 (Introduction to Software Development)

Winter 2018

08 Mathematics for MIS (H&S Ch. 3)

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08 Mathematics for MIS (H&S Ch. 3)

- Administrative details
- David Parnas
- Review of sets, relations and functions
- Review of logic
- Review of types, sets, sequence and tuples
- Multiple assignment statement
- Conditional rules
- Finite state machines
- Circle intersection example

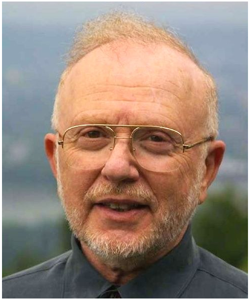
Administrative Details

- Assignment 1
  - Part 1: January 22, 2018
  - Partner Files: January 28, 2018
  - Part 2: January 31, 2018
- Questions on assignment?
- Tutorial this week also includes a mathematics review, with more examples
- Hoffman and Strooper (1995) provides a good review of discrete math for software specification

Avenue Posts (or E-mail) Advice

- First sentence is the question
- Be as clear as possible on what you want
- Provide supporting information - OS, VM?, steps taken, error message
- Consider a screen shot of the error message
- Thank the people that helped you
- For E-mail add identifying information
- For Avenue, consider unchecking "Include original post in reply" checkbox (under Settings)

Who is David Parnas?



A. A professor emeritus in the Faculty of Engineering
B. Started the McMaster Software Engineering program
C. Developed the idea of information hiding in modular design
D. One of the founding fathers of software engineering
E. All of the above

Mathematics for Module State Machines

- The material in this lecture should be review
- Shows the simple tools that can be used to build your MIS
- Shows the syntax we will use
- Follows
  - Hoffman and Strooper (1995), Chapter 3
  - Gries and Schneider

Sets, Relations and Functions

- A set is an unordered collection of elements
- A binary relation is a set of ordered pairs
- A function is a relation in which each element in the domain appears exactly once as the first component in the ordered pair

Sets

- An element either belongs to a set or it does not
- $x \in S$ versus $x \notin S$
- Defining a set
  - Enumerate $\{x_1, x_2, x_3, ..., x_n\}$
  - Logical condition (rule) from Gries and Schnieder notation $\{x : t | R : E\}$, where R is a predicate and E is an expression
  - An integer range $[2..4] = \{2, 3, 4\}$, $[7..4] = \{\}$
- Examples
  - $S = \{1, 7, 6\}$
  - $S = \{x : \mathbb{Z} | 1 \leq x \leq 4 : x\}$
  - $S = \{x : \mathbb{N} | 1 \leq 100 : x^2\}$
- Does $\{1, 7, 6\} = \{7, 1, 6\}$?

Sets

How would you write the set of points C inside a circle centered at the origin with a radius of r?

- What is the type of each element in the set?
- What is the dummy variable?
- What is the range?

Sets

How would you write the set of points C inside a circle centered at the origin with a radius of r?

$$C(r) = \{x, y : \mathbb{R} | x^2 + y^2 \leq r^2 : \langle x, y \rangle\}$$

- What if the circle is centered at (x_c, y_c)?

Relations

- Let $\langle x, y \rangle$ denote an ordered pair
  - $dom(R) = \{x | \langle x, y \rangle \in R\}$
  - $ran(R) = \{y | \langle x, y \rangle \in R\}$
- Defining a relation
  - Enumerate $\{< 0, 1 >, < 0, 2 >, < 2, 3 >\}$
  - Rule $\{x, y : \mathbb{Z} | x < y : \langle x, y \rangle\}$

Functions

- Let $\langle x, y \rangle$ denote an ordered pair
- Each element of the domain is associated with a unique element of the range
- Defining a function
  - Enumerate $\{\langle 0, 1\rangle, \langle 1, 2\rangle, \langle 2, 3\rangle\}$
  - Rule $\{x, y : \mathbb{Z} | y = x^2 : \langle x, y \rangle\}$
- Notation
  - $f(a) = b$ means $\langle a, b \rangle \in f$
  - $f(x) = x^2$
  - $f : T_1 \rightarrow T_2$
  - $\{x_1, x_2, y : \mathbb{Z} | y = x_1 + x_2 : \langle\langle x_1, x_2\rangle, y\rangle\}$
- Is $\{\langle 0, 1\rangle, \langle 0, 2\rangle, \langle 2, 3\rangle\}$ a function?
- Is $\{x, y : \mathbb{Z} | y^2 = x : \langle x, y \rangle\}$ a function?

Logic

- A logical expression is a statement whose truth values can be determined ($6 < 7$?)
- Truth values are either *true* or *false*
- Complex expressions are formed from simpler ones using logical connectives ($\neg$, $\wedge$, $\vee$, $\rightarrow$, $\leftrightarrow$)
- Truth tables
- Evaluation
  - Decreasing order of precedence: $\neg$, $\wedge$, $\vee$, $\rightarrow$, $\leftrightarrow$
  - Evaluate from left to right
  - Use rules of boolean algebra

Quantifiers

- Variables are often used inside logical expressions
- Variables have types
- A type is a set of values from which the variable can take its value
- Often quantify a logical expression over a given variable
  - Universal quantification
  - Existential quantification

Quantifiers Continued

- Bound variables appear in the scope of the quantifier
- Free variables are not bound to any quantifier
- Free variables in an expression often mean that we cannot determine the truth value of the expression
- Gries and Schneider use the following notation $(\forall x : t | R : P)$ where R is a predicate (for the range) and P is a predicate
- $\forall$ is applied to the values P for all x in t for which range R is true

Types, Sets, Sequence and Tuples

- A type is a set of values, so any precisely defined set is a type
- Primitive types are integer, boolean, character, string and real
- User-defined types
  - The set of values has to be given
  - Often use type constructors
- Useful type constructors
  - Set
  - Sequence
  - Tuple

Types

- Specify the type of a variable
  - $x_1, x_2, ..., x_n : T$
  - $x : integer$ or $x : \mathbb{Z}$
  - $a, b, c : string$
- Type definition
  - $T = d$
  - $float = real$
  - $colour = \{red, white, blue\}$
  - $testtype = \{uniaxial, biaxial, shear\}$
  - $x : testtype$
  - $motionT = \{forward, backward, stop\}$

Primitive Types

- Integer
  - $\{... -2, -1, 0, 1, 2, ...\}$
  - $+, -, \times, /$
  - $=, \neq$
  - $<, \leq, \geq, >$
- Real
  - $\{\mathit{all\ real\ numbers}\}$
  - $+, -, \times, /, \text{sin}(), \text{cos}(), \text{exp}()$ etc.
  - $=, \neq$
  - $<, \leq, \geq, >$

Primitive Types Continued

- Boolean type
  - $\{true, false\}$
  - $\neg$, $\wedge$, $\vee$, $\rightarrow$, $\leftrightarrow$
- Char type
  - Set of ASCII characters
  - Character values appear in quotes $'a'$, $'b'$, $'c'$, etc.
  - $=, \neq$

Primitive Types Continued

- String type
  - All finite sequences of characters
  - String constants are in double quotes $"abc"$
  - $s[i..j]$ is the substring of s from position i to position j
  - $s_1||s_2$ concatenates strings s_1 and s_2
  - $=, \neq$ for is equal and not equal
  - $\in, \notin$ for is member and not a member
  - $s[i]$ is the ith character of s
  - $|s|$ is the length of s
  - Positions in strings are zero relative

Sets

- A set is an unordered collection of elements of the same type
- Declare a set of type T as *set of* T
- Example
  - $T = \mathit{set\ of}\ \{\mathit{red}, \mathit{green}, \mathit{blue}\}$ defines type T as the power set of $\{\mathit{red}, \mathit{green}, \mathit{blue}\}$
  - $x : \mathit{set\ of\ integer}$
- What are some possible values for $x : \mathit{set\ of\ integer}$?

Operations on Sets

- $\cup$ union
- $\cap$ intersection
- $-$ difference
- $\times$ Cartesian product
- $=, \neq$ equal, not equal
- $\in, \notin$ member, non-member
- $|s|$ size of set s

Sequences

- A sequence is an ordered collection of elements of the same type
  - Elements can occur more than once
  - Sometimes referred to as a list
  - Similar to an array
- Declare a sequence of type T by *sequence of* T
- $< x_0, x_1, ..., x_n >$ for $n \geq 0$ for a sequence with elements $x_0, x_1, ..., x_n$
- $<>$ is the empty sequence
- Position in a sequence is zero relative

Sequences Continued

- Examples
  - $T = sequence\ of\ \{red, green, blue\}$ defines the type T as the set of all sequences of elements from $\{red, green, blue\}$
  - $x : sequence\ of\ integer$
- Fixed-length sequence of type T with length l
  - $sequence\ [l]\ of\ T$
  - l is a positive integer
  - $sequence\ [l_1, l_2, ..., l_n]\ of\ T$ is a shorthand for $sequence\ [l_1]\ of\ sequence\ [l_2]\ of ... sequence\ [l_n]\ of\ T$

Operations on Sequences

- $s[i..j]$ is the subsequence of s from position i to position j
- $s[i..j]$ is undefined if $i \notin [0..|s|-1] \vee j \notin [0..|s|-1]$
- $s_1||s_2$ concatenates sequences s_1 and s_2
- $=, \neq$ for is equal and not equal
- $\in, \notin$ for is member and not a member
- $s[i]$ is the ith element of s
- $s[i]$ is undefined if $i \notin [0..|s|-1]$
- $|s|$ is the length of s
- A string is a sequence of characters

Tuples

- A tuple is a collection of elements of possibly different types
- Each tuple has one or more fields
- Each field has a unique identifier called the field name
- Similar to a record or a structure
- To declare a tuple use
  - *tuple of* $(f_1 : T_1, f_2 : T_2, ..., f_n : T_n)$ with $n \geq 1$
  - f_i is the name of the ith field
  - T_i is the type of the ith field
  - *tuple of* $(f_1, f_2, ..., f_n : T)$ if all fields are of the same type

Example Tuples

- Examples
  - pair = tuple of (id: integer, val: string)
  - experimentT = tuple of (b$_{cond}$: bcondT, control: controlT)
  - pointT = tuple of (x: real, y: real)
- Define the value of a tuple by using an expression of the form
  - $< x_1, x_2, ..., x_n >$
  - $< 4,'' cat'' >$ is a value of type pair

Operations on Tuples

- $=, \neq$ equal, not equal
- $t.f$ is the value of field f of tuple t

Using Type Constructors

- $bcondT = \{uniaxial, biaxial, multiaxial, shear\}$
- $controlT = \{load\_controlled, displacement\_controlled\}$
- $experimentT = tuple\ of\ (b_{cond} : bcondT, control : controlT)$
- $experiment : experimentT$
- $directionT = \{clockwise, counterclockwise\}$
- $powerT = [MIN\_POWER...MAX\_POWER]$
- $motorT = tuple\ of\ (powerOn : Boolean,\ direction : directionT,\ powerLevel : powerT)$

Multiple Assignment Statement

- $v_1, v_2, ..., v_n := e_1, e_2, ..., , e_n$ with $n \geq 1$
- The v_is are distinct variables and each e_i is an expression of the same type as v_i
- Compute the values of all the expression e_i and then assign these values simultaneously
- Example
  - $x, y := 0, 10$
  - $x, y := 10, x$
  - $x, y := y, x$
- Convenient for defining the meaning of pieces of code
- Use as a function on the state space of a program

Uses of Conditional Rules

- To define the value of a function
- $min(x, y) = (x \leq y \Rightarrow x | x > y \Rightarrow y)$
- To define the meaning of a program
  - If $(x < y)$ then $z := x$ else $z := y$
  - $(x < y \Rightarrow z := x | x \geq y \Rightarrow z := y)$
  - $(x < y \Rightarrow x, y := x, y | x \geq y \Rightarrow x, y := y, x)$
- Conditional rules can be expressed in tables

Finite State Machines

- A FSM is a tuple $(S, s_0, I, O_E, O_O, T, E, C)$ where
- S is a finite set of states
- s_0 is the initial state in S $(s_0 \in S)$
- I is a finite set of inputs
- $T : S \times I \rightarrow S$ is the transition function
- O_E is a finite set of event outputs
- $E : S \times I \rightarrow O_E$ is the event output
- O_C is a finite set of condition outputs
- $C : S \rightarrow O_C$ is the condition output

Homework Answer for `intersect(c)`

out $:=?$

- What is the type of out?
- How would you calculate the output (in words)?
- `intersect(c)` knows the centre and radius of the current object – how do we get the centre and radius of `c`?
- What is the type of the set of points inside a circle?

CircleT ADT MIS: Syntax

Access Routine Syntax

Routine name	Inputs	Outputs	Exceptions
new CircleT	real, real, real	CircleT	
xc		real	
yc		real	
r		real	
area		real	
...	...	...	...
intersect	CircleT	Boolean	

State variables

xc: real

yc: real

r: real

Homework Answer for `intersect(c)`

intersect(c):

out : $=$
circlePts(xc, yc, r) $\cap$ circlePts(c.xc, c.yc, c.r) $\neq \emptyset$

circlePts(xc, yc, r): $\mathbb{R} \times \mathbb{R} \times \mathbb{R} \rightarrow$ set of $\mathbb{R}^2$

circlePts(xc, yc, r)
$\equiv \{x, y : \mathbb{R} | (x - xc)^2 + (y - yc)^2 \leq r^2 : \langle x, y \rangle\}$

Homework Answer for `intersect(c)`

out : =
$(\exists \langle x, y \rangle : \mathbb{R}^2 | (x - xc)^2 + (y - yc)^2 \leq r^2 :$
$(x - c.xc)^2 + (y - c.yc)^2 \leq (c.r)^2)$

out : $= \text{dist}(xc, yc, c.xc, c.yc) \leq (r + c.r)$

where
dist(x_1, y_1, x_2, y_2): $\mathbb{R} \times \mathbb{R} \times \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$

dist(x): $\equiv \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

Which version is most abstract?

Which version is closest to code?

References

- Hoffman and Strooper, *Software Design, Automated Testing and Maintenance*, International Thomson Computer Press, 1995.
- Gries and Schneider, *A Logical Approach to Discrete Math*, Springer, 1993.