SE 2AA4, CS 2ME3 (Introduction to Software Development)

Winter 2018

25 English To Math

Dr. Spencer Smith

Faculty of Engineering, McMaster University

March 12, 2018

25 English To Math

- Administrative details
- English and mathematics as languages
- Different world views
- Translating between languages
- Strategies
- Translator's glossary
- Exercise

Administrative Details

- Midterm
  - Grades on Avenue
  - Average around 70%
  - Will change to out of 29 for final grade calculation
- A3
  - Part 1 - Specification: due 11:59 pm Mar 12
  - Part 1 - Solution: Mar 18
  - Part 2 - Code: due 11:59 pm Mar 26
  - Do not delete wss comments
  - Look at minor diffs in se2aa4_cs2me3 repo for `spec.tex`
- A4
  - Your own design and specification
  - Due April 9 at 11:59 pm

Natural Languagues

- How many natural languages do you know?
- What natural languages do you know?

English and Mathematics as Languages

See Baber2002

- English is a language
- So is Mathematics
- Both have
  - Rules of grammar (syntax)
  - Semantics
- When writing in any language, pay attention to grammar and semantics. Get both right.

English and Mathematics: A Difference

- In English and other natural languages
  - Ambiguity desired, intentionally possible
  - Unambiguous statements almost impossible
- In Mathematics
  - Ambiguity not desired, intentionally prevented
  - Ambiguous statements almost impossible (even in probability theory, fuzzy logic)

Mathematics and Engineering

- Therefore, mathematics is the language of engineering

Correct Syntax for Mathematics

- Make sure the syntax of your mathematical expressions is correct
- Correct syntax does not guarantee correct semantics
- Incorrect syntax makes the mathematics ambiguous
- Example problems to watch for
  - Mismatch of types
    - $(\text{p.xcoord}() + \mathit{width}) \wedge \text{p.xcoord}()$
    - $\neg(\text{integer})$
    - Set of sequences accessed by $[i]$
    - $\forall (i : \mathbb{N} \wedge j : \mathbb{N} \ldots)$
  - Use of programming language notation in mathematics
    - Integer values instead of boolean
    - $\text{length} == \text{MAX\_SIZE}$
  - $(x > 7) = \mathit{true}$ instead of $(x > 7)$ (bad form)

Different World Views Question

- Do the following languages have a static or dynamic world view?
  - Imperative programming language?
  - Mathematics?
  - Object oriented programming language?
  - Functional programming language?

Different World Views

- English and other natural languages
  - Express both static and dynamic views
  - States and actions (verbs of being and action)
- Imperative programming languages
  - Primarily dynamic world view (changes)
- Functional programming languages
  - Static world view
- Mathematics
  - Static world view only
- Fundamental conceptual differences

Static Versus Dynamic Views

- These very different world views pose a conceptual hurdle for the translator
- The translator must bridge the gap between
  - Dynamic and static view of problem statement
  - Dynamic world view of imperative and OO programming and
  - Purely static world view of mathematics
- Not hard, but requires conscious attention

Translating Between Languages

- Lost in Translation
- Translating a statement from one language to another is a multistep (not single) process
  1. Statement in source language to a mental understanding of the meaning of the statement
  2. Reformulate mental understanding into target language view, concepts, culture
  3. Mental understanding of the meaning of the statement to a statement in the target language
- The first and last statement must mean the same

Translators

- Knowing two languages: not enough to translate
- A good translator knows well
  - The two languages
  - AND the subject being translated
  - AND how to translate
- These three things are different

Organization and Style

- When writing in English or any other natural language, one pays careful attention to
  - Organization of the essay, report, etc.
  - Style of expression
- When writing in Mathematics, to do the same:
  - Clear, complete, conscise
  - KISSS (Keep it Simple Sharp and Straightforward)
  - Understandable
  - Interesting

Strategies

- Understand the meaning of the original
- Obtain all needed information
- Close the gap between the English text and mathematics
- Divide and conquer (complexity)

Strategy: Understand the Original

- Describe specific instance of general problem
- Distinguish essentials from background
- Draw a diagram
- Express in intermediate or mixed language
- Identify object referred to
- Identify implicit (but false) "information"
- Identify missing information
- Identify relationships between essential objects
- Identify special cases

Strategy: Obtain all Needed Information

- Ask the author of the task description
- Identify gaps in the description of the task
- Identify implicit "information"
- Ask if implicit "information" may be assumed
- Identify data present and ask about related details
- Ask if missing information is really needed
- Read carefully, thoroughly, precisely

Strategy: Close Gap English – Mathematics

- Express implicit information explicitly
- Reduce vagueness and ambiguity
- Reword English text to be closer to mathematics (express in an intermediate, mixed language)

Strategy: Divide and Conquer

- Construct a table
- Distinguish between specific cases
- Introduce an auxiliary mathematics function
- Modularize
- Example
  - Find the location in D of a particular value *key*
  - What are the two cases?

Strategy: Divide and Conquer

- Construct a table
- Distinguish between specific cases
- Introduce an auxiliary mathematics function
- Modularize
- Example
  - Find the location in D of a particular value *key*
  - What are the two cases?
  - *key* in D, *key* not in D

Strategy: Divide and Conquer

- Construct a table
- Distinguish between specific cases
- Introduce an auxiliary mathematics function
- Modularize
- Example
  - Find the location in D of a particular value key
  - What are the two cases?
  - key in D, key not in D
  - How do you mathematically specify key is present in D?

Strategy: Divide and Conquer

- Construct a table
- Distinguish between specific cases
- Introduce an auxiliary mathematics function
- Modularize
- Example
  - Find the location in D of a particular value *key*
  - What are the two cases?
  - *key* in D, *key* not in D
  - How do you mathematically specify *key* is present in D?
  - $\vee(i : \mathbb{N} | 1 \leq i \leq |D| : D[i] = key)$ (Baber starts his list index at 1)

Strategy: Draw Diagrams, Describe Specific Instances of the Given Problem

- Graphical representations help understand the meaning of the message
- For specific instances, think of extreme cases first to simplify
  - $n = 0$
  - $n = 1$
  - $n = \inf$
- Think of a normal sized problem, usually something like $n \geq 3$
- You might want to write down truth tables

A Small Translator's Glossary

English	Mathematics
and, but	$\wedge$
or	$\vee$
for all, each, every, any	$\forall$, $\wedge$ series, universal quantification
for no, none	$\forall$, $\wedge$ series, universal quantification, with a negated assertion
there is (are), there exist(s), for some, at least one	$\exists$, $\vee$ series, existential quantification

A Small Translator's Glossary Continued

English	Mathematics
integer	... $\in \mathbb{Z}$
sorted	$\wedge_{i=0}^{n-2} A[i] \leq A[i+1]$, $\forall(i : \mathbb{N} \| 0 \leq i < n-1 : A[i] \leq A[i+1])$
if (when, whenever) ... then ...	... $\rightarrow$ ..., sometimes $\wedge$
search, find, equal, present	$=$
exchange, rearrange, different order, different sequence, merge, copy, sort	permutation

Your Translator's Glossary

- A professional translator compiles his/her own translation glossary
  - Over time
  - Based on own accumulated experience
- You should too!

Students with the same birthday

Consider a class with many students. The instructor bets the students that two or more students have the same birthday. Write a mathematical expression that is True if this condition is met, and False otherwise

- Identify all objects referred to in this message

Students with the same birthday

Consider a class with many students. The instructor bets the students that two or more students have the same birthday. Write a mathematical expression that is True if this condition is met, and False otherwise

- Identify all objects referred to in this message
- Class? Students? Many? Instructor? Birthday?
- Distinguish between essential objects and background information

Students with the same birthday

Consider a class with many students. The instructor bets the students that two or more students have the same birthday. Write a mathematical expression that is True if this condition is met, and False otherwise

- Identify all objects referred to in this message
- Class? Students? Many? Instructor? Birthday?
- Distinguish between essential objects and background information
- How would we model the relationship between a student and their birthday? A set? A type? A variable? A relation? A function?

Students with the same birthday

Consider a class with many students. The instructor bets the students that two or more students have the same birthday. Write a mathematical expression that is True if this condition is met, and False otherwise

- Identify all objects referred to in this message
- Class? Students? Many? Instructor? Birthday?
- Distinguish between essential objects and background information
- How would we model the relationship between a student and their birthday? A set? A type? A variable? A relation? A function?
- Using StudentT, C : StudentT and $\text{Bday}(s1) = \text{Bday}(s2)$ what is the predicate?

Students with the same birthday

Consider a class with many students. The instructor bets the students that two or more students have the same birthday. Write a mathematical expression that is True if this condition is met, and False otherwise

- Identify all objects referred to in this message
- Class? Students? Many? Instructor? Birthday?
- Distinguish between essential objects and background information
- How would we model the relationship between a student and their birthday? A set? A type? A variable? A relation? A function?
- Using StudentT, C : StudentT and $\text{Bday}(s1) = \text{Bday}(s2)$ what is the predicate?
- $\vee(s1, s2 : \text{StudentT} | s1 \in C \wedge s2 \in C \wedge s1 \neq s2 : \text{Bday}(s1) = \text{Bday}(s2))$

Exercise

Consider an array D with index values ranging from 1 to n. The subject of this example is part of a specification for a subprogram that will count how many times a particular given value occurs in the array D.

The goal of this exercise is to write a postcondition for the subprogram, relating the various relevant variables values when the search is complete.

Exercise Continued

Understand the task in the original language

- Identify objects referred to (look for nouns in the original English text)
- Identify missing information

Exercise Continued

Understand the task in the original language

- Identify objects referred to (look for nouns in the original English text)
- Array D, index value, times (count), particular given value, relevant variables's value
- Identify missing information

Exercise Continued

Understand the task in the original language

- Identify objects referred to (look for nouns in the original English text)
- Array D, index value, times (count), particular given value, relevant variables's value
- Identify missing information
- Names of variable for: index, times (count), particular given value

Exercise Continued

Understand the task in the original language

- Identify objects referred to (look for nouns in the original English text)
- Array D, index value, times (count), particular given value, relevant variables's value
- Identify missing information
- Names of variable for: index, times (count), particular given value
- Are there any other relevant variables?

Exercise

- Identify missing information
- Names of variable for
  - Index: assume *i*
  - Times (count): ask the author of the task, assume *count*
  - Particular given value: Ask the author of the task, assume *key*
  - Are there any other relevant variables? (no?)

Exercise Continued

- Close the gap between English text and mathematics
  - Reword the English text to be closer to mathematics
  - Use the English verb *count*

Exercise Continued

- The English verb *count* means, in programming language and in terms closer to mathematics, *add* 1
- But this is a dynamic (action) concept
- The corresponding static (state, relational) concept in mathematics is the function *addition with 1*, that is $+1$

Exercise Continued

- The occurrence of the particular given value in an array element in D
- $D[i] = key$
- A condition for the addition with 1
- The repetition over a variable number of index values suggests quantification with the function addition and with the argument 1
- $+(i :?|... \land D[i] = key : 1)$

Exercise Continued

Identify relationships between essential objects

- Array D, index value, particular given value: $D[i] = key$
- Combine the above with count ($+$ conditionally with 1)
- $count = +(i : \mathbb{N} | ... \leq i \leq ... \wedge D[i] = key : 1)$
- Range of i missing
- Refer to original English text: 1 to n
- $count = +(i : \mathbb{N} | 1 \leq i \leq n \wedge D[i] = key : 1)$

Exercise: New Glossary Entry

Now we have a new entry for our glossary

- count: $+(i : \mathbb{N} | ... \leq i \leq ... \wedge ... : 1)$, where the ... defines the range of the quantified variable and the condition for counting

Modelling Larger Components

- Baber focuses on translating low level mathematical expressions
- In our specification, we will also need to organize the information
- For larger problems, think about your types, Abstract Data Types and Abstract objects
- Interactively switch between types, ADTs, Abstract Objects and detailed mathematical spec
- Example
  - Modelling game state for Freecell
  - What are some potential types?
  - What are some mathematical expressions you will need?

Summary

- Knowledge of English and Mathematics is necessary but not sufficient to translate into Mathematics
- Knowledge of subject area also needed
- Translating skills needed
- The three are different

Summary

- Compile your own glossary
- Make intermediate steps and expressions conscious
- Modularize
- Organize systematically
- KISSS

Reference

- Baber, Robert L., *Translating English to Mathematics*, 2002